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again at  $U_s$ . Let the variable circle cut  $g$  again at  $H$ . Then as triangles  $QRU_s$  and  $SHU_s$  are similar:

$$\frac{U_s H}{U_s S} = \frac{U_s Q}{U_s R} = k, \text{ constant.}$$

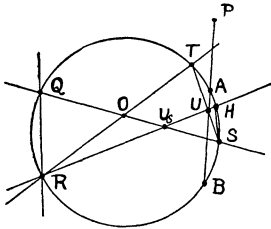


FIG. 2.

As the variable line  $ST$  moves parallel to itself, the segments it cuts on the fixed lines from  $U_s$  are proportional, so

$$\frac{U_s U}{U_s S} = k', \text{ constant.}$$

Thence

$$\frac{U_s H}{U_s U} = \frac{k}{k'} = c, \text{ constant.}$$

The procedure given is to draw  $PU$  arbitrarily from  $P$  to cut  $g$  at  $U$ ; to determine  $H$  on  $g$  from the constant ratio just proved; and then to construct the circle through  $QRH$  to cut  $PU$  in points of the curve.

## A GENERALIZATION OF THE STROPHOID.

By J. H. WEAVER, Ohio State University.

W. W. Johnson has given the following generalization for the strophoid.<sup>1</sup> Let  $A$  and  $B$  be two fixed points, and let two variable lines  $PA$  and  $PB$  make with  $AB$  angles  $\phi$  and  $\psi$ , respectively. Let  $\alpha$  be a constant angle and let  $P$  move so that

$$n\phi \pm m\psi = \alpha. \quad (1)$$

Then the locus of  $P$  is a strophoid. Equation (1) shows that there is associated with this set of curves a circle having a segment with  $AB$  as base in which the angle  $\alpha$  may be inscribed.

In the following discussion some curves are developed which have associated with them the three conic sections.

**Elliptic Case.** Let there be an ellipse  $E$  with major axis  $AB$ , and from  $A$  and  $B$  let variable lines  $AP$  and  $BP$  be drawn making angles  $\theta_1$  and  $\theta_2$  respectively with  $AB$ . Let  $AQ$  and  $BQ$  be so drawn as to make angles  $\pm m\theta_1$  and  $\pm n\theta_2$  with  $AB$ . When the locus of  $Q$  is the ellipse  $E$ , the locus of  $P$  is a curve whose equation may be developed as follows. (In this development we will consider  $m$  and  $n$  as positive integers and relatively prime to each other.)

The slope of  $AQ$  is  $\tan(m\theta_1)$  and of  $BQ$  is  $\tan(n\theta_2)$ . Then since  $Q$  is on  $E$  we have

$$\tan(m\theta_1) \cdot \tan(n\theta_2) = -b^2/a^2, \quad (2)$$

<sup>1</sup> "The Strophoids," *American Journal of Mathematics*, vol. 3, 1880, pp. 320-325. See also G. Loria, *Spezielle algebraische und transcendente ebene Kurven*, Berlin, vol. 1, 1910, p. 73. This class of curves includes the sextrix curves as a subclass. See Loria, l.c., p. 390.

where the equation of  $E$  is  $x^2/a^2 + y^2/b^2 = 1$ . Then using the same axes, the equations of  $AP$  and  $BP$  are

$$(AP) \quad y = (x + a) \tan \theta_1, \quad (3)$$

$$(BP) \quad y = (x - a) \tan \theta_2. \quad (4)$$

Eliminating  $\theta_1$  and  $\theta_2$  from (2), (3) and (4) we have

$$\begin{aligned} & a^2 \left[ \binom{m}{1} y(x + a)^{m-1} - \binom{m}{3} y^3(x + a)^{m-3} \dots \right] \\ & \cdot \left[ \binom{n}{1} y(x - a)^{n-1} - \binom{n}{3} y^3(x - a)^{n-3} \dots \right] \\ & + b^2 \left[ (x + a)^m - \binom{m}{2} y^2(x + a)^{m-2} \dots \right] \left[ (x - a)^n - \binom{n}{2} y^2(x - a)^{n-2} \dots \right] \\ & = 0, \end{aligned} \quad (5)$$

an equation of degree  $m + n$ .

We will designate the curve (5) as  $C_{m,n}$ . The curve  $C_{m,n}$  has  $m - n$  asymptotes. These pass through the same point<sup>1</sup> if  $a = b$ . If  $m - n$  is odd, there will be one asymptote parallel to the  $y$  axis. Its equation will be

$$x = \frac{nb^2 + ma^2}{nb^2 - ma^2}a, \quad \text{or} \quad x = \frac{na^2 + mb^2}{na^2 - mb^2}a,$$

the former when  $m$  is even and  $n$  odd, the latter when  $m$  is odd and  $n$  even.

**THEOREM I.** *The curve  $C_{m,n}$  has an  $m$ -tuple point at  $A$  and an  $n$ -tuple point at  $B$ , and the tangents to the curve at each of these points make equal angles with each other.*

This theorem may be readily proved in the usual way.

**THEOREM II.** *The  $r$ th polar of  $B$  with respect to  $C_{m,n}$  is  $C_{m-r,n}$  ( $r \leq m$ ), and of  $A$  is  $C_{m,n-r}$  ( $r \leq n$ ).*

By substituting  $-b^2$  for  $b^2$  we obtain the hyperbolic case.

**Parabolic Case.** Let there be given a parabola and let there be drawn through the vertex a line making an angle  $n\phi$  with the axis and cutting the parabola in the point  $Q$ . Through  $Q$  draw a line parallel to the axis, and let this line be cut in the point  $P$  by a line through the vertex of the parabola and making an angle  $\phi$  with the axis of the parabola. Then the locus of  $P$  is a parabolic strophoid. Its equation may be determined as follows: Let  $O$  be the origin, and let the vertex of the parabola be at the origin, and its axis the  $x$ -axis. Then the equations of the parabola and of the lines  $OQ$  and  $OP$  are

$$y^2 = 4ax, \quad (6)$$

$$(OQ) \quad y = x \tan (n\phi), \quad (7)$$

$$(OP) \quad y = x \tan \phi. \quad (8)$$

<sup>1</sup> See Loria, l.c., p. 392.

From (6) and (7) we may find the equation of the line through  $Q$  parallel to the  $x$ -axis, and eliminating  $\tan \phi$  from this equation and (8) we get for the equation of the curve

$$y = \frac{4a \left[ x^n - \binom{n}{2} x^{n-2} y^2 \dots \right]}{\binom{n}{1} x^{n-1} y - \binom{n}{3} x^{n-3} y^3 \dots} \quad (9)$$

The curve (9) has an  $n$ -tuple point at the origin, and the tangents at this point make equal angles with each other.

Since (9) is of degree  $n + 1$  we will designate it as  $C_{n+1}$ .

**THEOREM III.** *The  $(r + 1)$ th polar of the point at infinity on the axis of the parabola  $y^2 = 4ax$  with respect to  $C_{n+1}$  is  $C_{n-r}$ .*

This may be proved by finding the polars in the ordinary way.

**A Special Elliptic Case.** In equation (5) let  $m = 1$  and  $n = 2$  and we obtain the cubic

$$b^2 y^2 (x + a) - 2a^2 y^2 (x - a) - b^2 (x + a)(x - a)^2 = 0. \quad (10)$$

**THEOREM IV.** *If in the curve (10) a line is drawn through  $A$  cutting the curve in the points  $P$  and  $P'$ , and the ellipse in  $C$ , then the tangents to (10) at  $P$  and  $P'$  and to  $E$  at  $C$  are concurrent.*

*Proof:* Draw through  $A$  another line cutting (10) in the points  $P_1$  and  $P'_1$  and  $E$  in  $C_1$ . From the construction of the curve (10) it follows that  $(AC, PP')$  and  $(AC_1, P_1 P'_1)$  are harmonic ranges. Since these ranges have a point  $A$  in common they are perspective. Therefore  $PP_1$ ,  $CC_1$  and  $P'P'_1$  are concurrent. If now we consider the lines  $AP$  and  $AP_1$  to become coincident, the lines  $PP_1$ , etc., become tangents.<sup>1</sup>

The two following theorems may also be proved without difficulty:

**THEOREM V.** *The first polar of  $A$  with respect to (10) is the ellipse  $E$ .*

**THEOREM VI.** *The locus of the point of concurrence of the tangents in theorem IV is a cuspidal cubic, having  $B$  as cusp and the asymptote of (10) as asymptote.*

If in (10)  $a = b$ , the curve is the right strophoid, and if  $b^2 = a^2/2$  it is the folium of Descartes.

**A Special Parabolic Case.** Let  $n = 2$  in (9) and it becomes

$$2a(x^2 - y^2) = xy^2. \quad (11)$$

**THEOREM VII.** *Let a line be drawn parallel to the axis of the parabola  $y^2 = 4ax$  cutting the parabola in the point  $C$  and the cubic (11) in the points  $A$  and  $B$ , and let tangents be drawn to (11) at the points  $A$  and  $B$  and to the parabola at  $C$ . These tangents are concurrent.*

The proof is similar to that of theorem IV.

**THEOREM VIII.** *The locus of the point of concurrence of the tangents in theorem*

<sup>1</sup> See Bassett, *An Elementary Treatise on Cubic and Quartic Curves*, Cambridge, 1901, p. 67.

*VII is a cissoid which has its cusp at the vertex of the parabola and whose asymptote is the asymptote of (11).*

THEOREM IX. *The parabola  $y^2 = 4ax$  is the curvilinear diameter of the cubic (11).*

The proofs of theorems VIII and IX offer no difficulty and are therefore omitted.

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## HONOR TO PROFESSOR E. H. MOORE.

By H. E. SLAUGHT, University of Chicago.

At the two hundred twenty-second regular meeting of the American Mathematical Society, held at the University of Chicago April 14, 15, 1922, there was celebrated the twenty-fifth anniversary of the founding of the Chicago Section of this Society. The history of this Chicago Section is almost coincident with that of the University of Chicago where the great majority of its forty-nine meetings have been held. These meetings, which began in 1896 with the reading of fourteen scientific papers at the first informal session, gradually increased in attendance and importance until they were recognized by the Society as of co-ordinate standing with those of the parent organization in New York and were officially designated as regular Western meetings of the Society. The one just held was the seventeenth and largest of this kind, over one hundred members being in attendance.

The young and vigorous department of mathematics of the University of Chicago in 1896, with its remarkable trio of leaders, Professors E. H. Moore, Oskar Bolza, and Heinrich Maschke, naturally assumed the important rôle of leadership in fostering mathematical research in the Middle West, as reflected in the phenomenal growth of the Chicago Section. To those who know Professor Moore scientifically and personally, it is no surprise that he at once became, and still remains, the leader of leaders in this great work. It is universally recognized that he stands quite alone as regards the scope and strength of his influence on the development of mathematics in America, not only through his own researches but also through his impress upon the hundred and more men and women who have gone out with the Chicago doctorate in mathematics, upon the hundreds of Chicago masters and other graduate students, and upon all others who have directly or indirectly come within his dynamic presence and captivating friendship.

It was a foregone conclusion that any celebration of the last quarter of a century of mathematical activity in this country would center about Professor Moore, and hence a committee of his former students began more than a year ago to consider what kind of a testimonial would be most appropriate to present to him on this occasion. It was at once decided that none of the ordinary forms of gold or silver gifts, nor even a painted portrait, would adequately express the sentiments of his grateful admirers. They sought rather some token which